ECONASSETS GROUP, LLC

FINANCIAL INTELLIGENCE & QUANTITATIVE RISK ENGINEERING

Deconstructing Mortgage Prepayment Options: The Market Void in Direct Derivatives, Structural Convexity Dilemmas, and Practitioner Synthetic Replication Engines

Why Liquid Direct Prepayment Options Do Not Exist and How Institutional Desks Construct Multi-Asset Convexity and Swaption Overlay Hedges

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Abstract: Every standard fixed-rate residential mortgage contains an unpriced, uncollateralized American call option granted to the borrower: the perpetual right to prepay outstanding debt principal at par ($100% of UPB) without penalty. In aggregate, this embedded option generates the largest single concentration of negative convexity ($\Gamma \ll 0$) and duration extension risk in global fixed income, exceeding $12 trillion in securitized debt. Paradoxically, despite this staggering systemic risk, there is no liquid, standardized, directly traded mortgage prepayment option market in the global financial system.

This working paper examines the fundamental structural reasons for this market void. We deconstruct the friction mechanisms that preclude standardized mortgage option trading: path-dependent retail behavioral heterogeneity, refinancing burnout, primary-to-secondary mortgage rate basis wedges, and the severe liquidity deficiencies of short-dated TBA options. We then formalize how institutional market participants—mortgage portfolio managers, MSR servicers, GSEs, and bank ALM desks—synthetically replicate and immunize prepayment optionality. We contrast pure dynamic linear delta hedging (and its substantial "buy-high/sell-low" whipsaw slippage) against multi-asset non-linear structures combining SOFR OIS swaps, OTC receiver/payer swaptions, Constant Maturity Swap (CMS) spread options, and IO/PO structuring. A complete, production-grade Python simulation engine in the Appendix demonstrates the quantitative failure of linear delta hedging and the mathematical formulation of a joint delta-gamma-vega immunized swaption overlay.

Contents

1 Introduction: The $12 Trillion Embedded Prepayment Option

The U.S. residential mortgage market is uniquely defined by a government-sponsored structural feature: the standard 30-year fixed-rate mortgage grants the homeowner an unrestricted, cost-free (or minimal transaction cost) American call option to retire their loan balance at par (100% of Unpaid Principal Balance, UPB) at any time prior to maturity [1, 4].

When an investor purchases a Mortgage-Backed Security (MBS), such as a Fannie Mae, Freddie Mac, or Ginnie Mae pass-through, the investor is economically long a default-free fixed-income bond and **short an uncollateralized American call option held by thousands of heterogeneous retail borrowers**:

$$P_{\text{MBS}}(y) = P_{\text{non-callable}}(y) - C_{\text{prepay}}\left(y, \sigma_y, \text{Burnout}_t, \text{WAC} - R_t\right) \tag{1}$$

where:

- $P_{\text{non-callable}}(y)$ is the theoretical present value of a non-callable amortizing cash flow stream discounted at prevailing market yields y.
- $C_{\text{prepay}}(\cdot)$ is the latent economic value of the borrower's embedded prepayment option.
- $\text{WAC} - R_t$ is the refinancing gross incentive (the pool's Gross Weighted Average Coupon minus current market borrowing rates).

1.1 1. The Geometry of Negative Convexity

In standard option-free fixed income (e.g., U.S. Treasuries or plain corporate bonds), the price-yield relationship exhibits **positive convexity** ($\frac{\partial^2 P}{\partial y^2} > 0$): as yields fall, price increases at an accelerating rate; as yields rise, price falls at a decelerating rate. Positive convexity is structurally favorable to the asset owner.

In mortgage-backed securities, the short call option completely subverts this relationship, generating severe **negative convexity** across the core refinancing coupon stack:

$$\text{Convexity} = \frac{1}{P}\frac{\partial^2 P}{\partial y^2} < 0 \tag{2}$$

The Asymmetric Mechanics of Negative Convexity:

1. **Falling Interest Rate Environment (Contraction Risk):** When market interest rates decline, bond prices should theoretically appreciate. However, falling rates incentivize homeowners to exercise their call options by refinancing into cheaper loans. The MBS investor receives their principal returned prematurely at par (100), truncating price upside and forcing reinvestment into lower-yielding assets. The duration of the MBS collapses (e.g., from 5.5 years to 1.2 years).
2. **Rising Interest Rate Environment (Extension Risk):** When market interest rates surge, homeowners rationally hold onto their low-rate mortgages. Refinancings drop to near-zero turnover baselines. Expected prepayments evaporate, cash flows slow to a trickle, and the maturity of the security dramatically lengthens. The duration of the MBS substantially extends (e.g., from 3.0 years to 8.5 years) precisely when the investor wants the shortest possible duration to avoid capital losses.

Figure 1: Price-Yield Curvature Distortion in Option-Embedded Mortgage Collateral

The mortgage investor is systematically exposed to the worst of both worlds: capped upside during rallies and leveraged downside during selloffs.

2 The Market Paradox: Why There Is No Liquid Direct Mortgage Option Market

Given that the mortgage market represents over $12 trillion in outstanding debt and generates massive negative convexity exposures across global banks, pension funds, insurance companies, and money managers, one would naturally anticipate the existence of a hyper-liquid, standardized, exchange-traded or OTC derivatives market in **direct mortgage prepayment options**.

Yet, no such direct market exists. Institutional hedgers cannot simply log onto an exchange or call a broker to buy a standardized "3-Year Prepayment Cap" or "5-Year Prepayment Put Option" linked directly to collateral pool prepayments. To understand this structural market failure, we must analyze five fundamental friction barriers:

2.1 1. Retail Behavioral Heterogeneity vs. Rational Option Theory

Financial derivatives pricing models (e.g., Black-Scholes, Cox-Ross-Rubinstein, Heath-Jarrow-Morton) operate on the bedrock assumption of **rational, frictionless, arbitrage-free exercise**: if an option is 1 basis point in the money at expiration, it is exercised with 100% certainty.

Mortgage prepayment options violate every assumption of rational option theory [2, 3]:

- **Bounded Rationality and Friction:** A borrower does not refinance the moment prevailing rates fall below their note rate. Refinancing requires out-of-pocket closing costs ($3,000–$6,000), appraisal fees, time-consuming loan documentation, and financial literacy.
- **Credit, Equity, and Documentation Rationing:** Even when refinancing is economically optimal, millions of borrowers cannot exercise their option due to macro credit rationing: negative equity ($LTV > 80\%$), insufficient debt-to-income (DTI) ratios, or damaged credit scores.
- **Non-Economic Prepayments:** Conversely, millions of borrowers prepay mortgages that are deeply out-of-the-money (e.g., prepaying a 3.0% mortgage when market rates are 7.0%) due to exogenous personal lifecycle events: divorce, death, job relocation, retirement downsizing, or home sales.

Because the exercise function is behavioral, sociological, and non-deterministic, market makers cannot establish a unique arbitrage-free hedging portfolio for a direct prepayment derivative.

2.2 2. Path-Dependence and the "Burnout" Phenomenon

Standard financial options are memoryless: the value of a call option on Apple stock depends on the current stock price, strike, volatility, and time to maturity, regardless of the historical path the stock took to get there.

Mortgage prepayment options exhibit extreme **path-dependence** known as **Burnout** [1, 6]:

$$\text{Prepayment Speed}_t = f\Big(\text{Incentive}_t\Big) \times \exp\left(-\delta \sum_{s=1}^{t-1} \text{Incentive}_s^+\right) \tag{3}$$

Consider two MBS pools with an identical current coupon of 6.50%:

- **Pool A (Unseasoned / Virgin Collateral):** Has never experienced a refinancing rally.
- **Pool B (Heavily Seasoned / Burned-Out Collateral):** Has survived two previous rate refinancing waves. All hyper-sensitive, financially sophisticated, high-balance borrowers have already refinanced out of the pool. The remaining borrowers are historically inert or credit-impaired.

If market mortgage rates drop to 5.0%, Pool A will experience a surging prepayment rate of 45% CPR, while Pool B will barely reach 12% CPR. Standardized derivatives contracts cannot easily define a single fungible underlying asset when collateral prepayment sensitivity is contaminated by path-dependent historical survivor bias.

2.3 3. The Primary-to-Secondary Mortgage Rate Basis Wedge

A direct mortgage prepayment option depends on whether the retail homeowner chooses to refinance. Homeowners do not refinance based on wholesale interbank SOFR swap rates or Treasury yields; they refinance based on the **retail primary mortgage rate** offered by retail mortgage lenders:

$$R_{\text{primary}}(t) = Y_{\text{Treasury}}(t) + \text{Swap Spread}(t) + \text{OAS}_{\text{secondary}}(t) + \text{G-Fee} + \text{Primary-Secondary Spread}(t) \quad (4)$$

The **Primary-to-Secondary Spread** represents the gross profit margin (gain-on-sale) captured by retail mortgage originators. During massive rate rallies (such as March–April 2020), wholesale capital market yields collapse overnight. However, mortgage originators become completely swamped with loan applications, exhaust their operational underwriting capacity, and intentionally **widen primary mortgage rates** by 100 to 150 basis points to throttle application volume [11].

Consequently, even though capital market rates fall, retail mortgage rates lag significantly. A direct prepayment option tied to wholesale interest rates would suffer massive model basis breakdowns during the exact market crises when protection is needed most.

2.4 4. The Severe Structural Deficiencies of TBA Options

The over-the-counter and exchange-traded To-Be-Announced (TBA) mortgage options market represents the closest existing direct derivative. However, institutional hedgers find TBA options fundamentally inadequate for systemic balance sheet hedging:

1. **Extreme Tenor Truncation:** Liquid TBA options are restricted almost exclusively to ultra-short expirations: 1-month, 2-month, and rarely 3-month tenors. Institutional hedgers managing 30-year mortgage pipelines and multi-year MSR portfolios require 2-year, 5-year, and 10-year option coverage.

2. **Prohibitive Roll Costs and Theta Decay:** Rolling 1-month TBA options over a 5-year hedging horizon generates heavy cumulative premium bleed (Θ_{decay}) and continuous dealer bid-ask slippage.

3. **Cheapest-to-Deliver (CTD) and Delivery Frictions:** Exercising a physical TBA call option results in the delivery of physical MBS pools. Market makers deliver the lowest-quality, fastest-prepaying, or least-desirable pools permitted under SIFMA good delivery guidelines, introducing adverse selection into the settlement process.

4. **Illiquidity of Non-ATM Strikes:** Trading depth is concentrated almost entirely in At-The-Money (ATM) options on current-coupon TBAs. Out-of-The-Money (OTM) wing strikes for deep tail-risk protection suffer from wide bid-ask spreads and limited dealer capacity.

3 How Practitioners Construct Synthetic Convexity Hedges

Faced with the complete absence of a liquid, long-dated direct mortgage option market, quantitative fixed-income practitioners cannot simply buy off-the-shelf insurance. Instead, they must **engineer synthetic hedges** by combining liquid linear and non-linear capital market derivatives.

Table 1: Structural Failure Modes of Direct Mortgage Prepayment Option Derivatives

Friction Mechanism	Market Reality in Mortgage Collateral	Impact on Direct Option Viability
Exercise Rationality	Boundedly rational, credit-constrained, lifestyle-driven	Precludes standard no-arbitrage Black-Scholes pricing
Path-Dependence	Burnout, media awareness lag, seasoning queues	Contract requires tracking full historical collateral rate path
Underlying Basis	Primary-Secondary spread decouples during rallies	Option fails to pay off when originator capacity limits bind
Derivative Tenors	Maximum liquid TBA option tenor is 1–3 months	Incapable of hedging multi-year asset-liability balance sheets
Collateral Fungibility	Fast-paying vs slow-paying story pools within same coupon	Cheapest-to-deliver adverse selection destroys contract par

3.1 1. The Failure of Static Linear Delta Hedging (The Portfolio Insurance Trap)

The most intuitive approach to hedging an MBS portfolio is dynamic linear delta hedging using 10-Year Treasury futures or SOFR interest rate swaps.

Let $D_{\text{MBS}}(y)$ be the effective duration of the mortgage asset. To establish delta neutrality, the hedger enters into a payer swap with notional N_{swap}:

$$N_{\text{swap}} = -N_{\text{MBS}} \cdot \frac{D_{\text{MBS}}(y)}{D_{\text{swap}}} \tag{5}$$

While delta neutrality holds for an infinitesimal rate shift $dy \to 0$, it breaks down significantly across finite rate moves due to negative convexity:

$$\Delta P_{\text{MBS}} \approx -D_{\text{MBS}} \cdot P \cdot \Delta y + \frac{1}{2}\Gamma_{\text{MBS}} \cdot P \cdot (\Delta y)^2 \tag{6}$$

$$\Delta V_{\text{swap}} \approx -D_{\text{swap}} \cdot N_{\text{swap}} \cdot \Delta y + 0 \quad (\text{since linear swaps have } \Gamma_{\text{swap}} \approx 0) \tag{7}$$

Net portfolio change under a linear delta hedge is dominated by unhedged negative convexity:

$$\Delta \Pi_{\text{net}} = \Delta P_{\text{MBS}} + \Delta V_{\text{swap}} \approx \frac{1}{2}\Gamma_{\text{MBS}} \cdot P \cdot (\Delta y)^2 \ll 0 \tag{8}$$

Because $\Gamma_{\text{MBS}} < 0$, **the hedged portfolio loses money regardless of whether interest rates rise or fall!**

- If rates fall 100 bps, the mortgage price caps while the payer swap loses full duration value $\to$ Net Loss.
- If rates rise 100 bps, mortgage duration extends, generating massive mortgage losses that exceed the linear swap gain $\to$ Net Loss.

To maintain delta neutrality, the manager must continuously rebalance the linear hedge: selling Treasuries/swaps after rates have already risen (selling low) and buying Treasuries/swaps after rates have already fallen (buying high). In a volatile, mean-reverting market, this continuous dynamic rebalancing bleeds massive capital—a phenomenon identical to the 1987 portfolio insurance breakdown.

3.2 2. Synthetic Volatility & Convexity Hedging via SOFR Swaptions

To neutralize negative convexity without suffering linear whipsaw decay, practitioners turn to the deep, liquid market in **OTC Interest Rate Swaptions** on the SOFR swap curve.

A swaption is an option to enter into an interest rate swap at a designated strike rate K:

- **Payer Swaption:** The right to pay fixed (acts as a put option on bond prices; gains value when interest rates rise).
- **Receiver Swaption:** The right to receive fixed (acts as a call option on bond prices; gains value when interest rates fall).

Because swaptions possess positive gamma ($\Gamma_{\text{swaption}} > 0$) and positive vega ($\mathscr{V}_{\text{swaption}} > 0$), hedgers use them to directly cancel the negative gamma and short vega of the embedded mortgage prepayment option:

$$\Gamma_{\text{total}} = \Gamma_{\text{MBS}} \cdot P_{\text{MBS}} + \sum_k N_{\text{swaption},k} \cdot \Gamma_{\text{swaption},k} = 0 \tag{9}$$

$$\mathscr{V}_{\text{total}} = \mathscr{V}_{\text{MBS}} + \sum_k N_{\text{swaption},k} \cdot \mathscr{V}_{\text{swaption},k} = 0 \tag{10}$$

Practitioner Swaption Overlay Structures:

1. **Receiver Swaptions (Contraction Immunization):** Hedgers purchase deep Out-of-The-Money (OTM) receiver swaptions (e.g., 1y10y or 2y10y receivers). If interest rates collapse, the surging value of the receiver swaption offsets the price-capping of the mortgage pool.
2. **Payer Swaptions (Extension Immunization):** Hedgers purchase OTM payer swaptions to protect against duration extension during rapid rate selloffs.
3. **Swaption Straddles and Strangles:** Purchasing both receiver and payer swaptions creates a pure long-convexity volatility bubble that insulates the MBS book against two-sided curve volatility.
4. **Collars and Ratio Structures:** To finance the high upfront premium costs of buying swaption gamma, desks execute collars: buying an ATM receiver swaption and simultaneously selling an OTM payer swaption, trading away extreme tail extension gains to eliminate out-of-pocket premium drag.

3.3 3. Synthetic Structuring via Interest-Only (IO) and Principal-Only (PO) Strips

In addition to capital market derivatives, practitioners utilize mortgage-native synthetic structuring by partitioning MBS cash flows into stripped mortgage-backed securities:

- **Principal-Only (PO) Strips:** PO investors receive only the principal cash flows. POs are purchased at a steep discount to par. When prepayments accelerate, principal is returned early at 100% of par, generating massive capital gains. A PO security is essentially a **pure leveraged long prepayment call option**.
- **Interest-Only (IO) Strips:** IO investors receive only the interest cash flows generated by the remaining balance. When prepayments surge, the underlying loan balance evaporates, extinguishing future interest payments. Consequently, IOs possess **extreme negative duration and massive positive convexity in high-rate regimes**.

By pairing standard pass-through MBS with custom allocations of IO and PO strips or CMO Planned Amortization Class (PAC) bonds, portfolio managers synthetically reshape their net convexity profile before deploying derivatives.

3.4 4. Constant Maturity Swap (CMS) Spread Options

Because mortgage prepayment waves are strongly correlated with yield curve slope (a flat or inverted yield curve suppresses refinancings; a steep curve accelerates them), macro mortgage desks trade **CMS Spread Options** (e.g., options on the 2Y–10Y or 10Y–30Y SOFR swap curve slope) to hedge macro prepayment regime shifts.

4 Mathematical Formulation of the Joint Delta-Gamma-Vega Hedge

To establish a mathematically rigorous hedging architecture, we formulate the joint optimization problem executed daily by institutional prepayment risk desks.

4.1 1. Second-Order Multi-Factor Taylor Expansion

Let the value of an MBS portfolio depend on prevailing interest rate levels y, term-structure curvature, and implied swaption volatility σ:

$$\Delta P = \frac{\partial P}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 P}{\partial y^2}(\Delta y)^2 + \frac{\partial P}{\partial \sigma}\Delta\sigma + \frac{\partial P}{\partial t}\Delta t \tag{11}$$

Expressing this in standard institutional risk Greeks:

$$\Delta P = -\mathrm{DV01}_{\mathrm{MBS}} \cdot (10^4 \cdot \Delta y) + \frac{1}{2}\mathrm{Gamma01}_{\mathrm{MBS}} \cdot (10^4 \cdot \Delta y)^2 + \mathrm{Vega}_{\mathrm{MBS}} \cdot \Delta\sigma - \Theta_{\mathrm{decay}} \cdot \Delta t \tag{12}$$

4.2 2. Formulating the Synthetic Immunization Matrix

To neutralize Delta, Gamma, and Vega simultaneously, the desk selects three liquid hedging instruments:

1. Instrument 1: Standard 10-Year SOFR Payer Swap (linear delta hedge; zero gamma/vega).
2. Instrument 2: 1y10y Receiver Swaption (non-linear hedge; high positive gamma and vega).
3. Instrument 3: 1y10y Payer Swaption (non-linear hedge; complements gamma while balancing vega).

The portfolio risk vector $\mathbf{G}_{\mathrm{MBS}} = \begin{bmatrix} \mathrm{DV01}_{\mathrm{MBS}} \\ \mathrm{Gamma01}_{\mathrm{MBS}} \\ \mathrm{Vega}_{\mathrm{MBS}} \end{bmatrix}$ is immunized by solving the linear algebraic system:

$$\begin{bmatrix} \mathrm{DV01}_{\mathrm{swap}} & \mathrm{DV01}_{\mathrm{rec\_swpn}} & \mathrm{DV01}_{\mathrm{pay\_swpn}} \\ 0 & \mathrm{Gamma01}_{\mathrm{rec\_swpn}} & \mathrm{Gamma01}_{\mathrm{pay\_swpn}} \\ 0 & \mathrm{Vega}_{\mathrm{rec\_swpn}} & \mathrm{Vega}_{\mathrm{pay\_swpn}} \end{bmatrix} \begin{bmatrix} N_{\mathrm{swap}} \\ N_{\mathrm{rec\_swpn}} \\ N_{\mathrm{pay\_swpn}} \end{bmatrix} = - \begin{bmatrix} \mathrm{DV01}_{\mathrm{MBS}} \\ \mathrm{Gamma01}_{\mathrm{MBS}} \\ \mathrm{Vega}_{\mathrm{MBS}} \end{bmatrix} \tag{13}$$

Because the upper-left block for the linear swap contains zero gamma and vega, the matrix is block upper-triangular:

1. The non-linear swaption notionals ($N_{\mathrm{rec\_swpn}}, N_{\mathrm{pay\_swpn}}$) are solved first to simultaneously extinguish mortgage negative gamma and vega.
2. The linear swap notional (N_{swap}) is solved second to absorb the residual net delta contributed by the swaptions and the underlying mortgage pool.

This decoupled solution guarantees complete second-order price immunization across both rate shocks and volatility shifts.

5 Conclusion

The absence of a direct, liquid mortgage prepayment option market is not an accident of financial history; it is the inevitable consequence of structural retail behavioral frictions, path-dependent burnout, primary-secondary operational basis wedges, and the physical delivery constraints of the TBA forward mechanism.

Because mortgage prepayment options cannot be directly procured in OTC or exchange markets, institutional risk desks must operate as financial engineers—synthesizing long-dated non-linear convexity overlays through sophisticated combinations of SOFR OIS swaps, swaptions, and structured strips. Mastering this multi-asset synthetic replication engine is the defining determinant of survival and profitability in institutional mortgage banking, balance sheet risk management, and structured credit investing.

Recommended Further Reading

For readers seeking to explore non-linear derivative replication, interest rate swaptions, and volatility surface modeling in greater depth, the following standard textbooks are recommended:

1. **Hull, J. C. (2018).** *Options, Futures, and Other Derivatives* (10th ed.). Pearson.
   Pedagogical Focus: The universally recognized benchmark textbook on derivative pricing. Chapters 28 and 29 provide comprehensive coverage of Black's model for swaptions, interest rate caps/floors, mortgage prepayment convexity, and multi-Greek dynamic immunization architectures.

2. **Tuckman, B., & Serrat, A. (2011).** *Fixed Income Securities: Tools for Today's Markets* (3rd ed.). John Wiley & Sons.
   Pedagogical Focus: Renowned for practitioner rigor in decomposing non-linear mortgage risks, constructing swaption straddles, and implementing synthetic convexity replication across complex institutional portfolios.

3. **Gatheral, J. (2006).** *The Volatility Surface: A Practitioner's Guide*. John Wiley & Sons.
   Pedagogical Focus: Delivers deep mathematical insight into implied volatility surfaces, swaption skew/smile dynamics, local volatility models, and the hedging of high-order non-linear exposures.

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A Standalone Python Engine: Prepayment Convexity Simulation & Swaption Overlay Hedge

The following production Python module simulates the embedded American prepayment option of an MBS pool across interest rate shocks, quantifies the severe performance failure of static linear delta hedging, and demonstrates the mathematical formulation of a joint SOFR Swap + Swaption synthetic convexity hedge.

```
"""
MBS Prepayment Option and Synthetic Convexity Replication Engine
EconAssets Group, LLC - Quantitative Research Series

Features:
1. Models the non-linear price, duration, and negative convexity of an MBS pool
   subject to embedded borrower prepayment options.
2. Demonstrates the failure of static linear delta hedging (portfolio insurance bleed).
3. Constructs a synthetic multi-asset hedge combining SOFR OIS Swaps (Delta) and
   OTC Swaptions (Gamma/Convexity/Vega).
4. Evaluates net hedged portfolio performance across rate rally, selloff, and whipsaw scenarios.
import numpy as np
import pandas as pd
import math

def norm_cdf(x):
    """Standard normal cumulative distribution function."""
    return 0.5 * (1.0 + np.vectorize(math.erf)(x / np.sqrt(2.0)))

def norm_pdf(x):
    """Standard normal probability density function."""
    return np.exp(-0.5 * x**2) / np.sqrt(2.0 * np.pi)

class MortgagePrepaymentPool:
    """
    Models an MBS pool with embedded borrower prepayment options.
    """
    def __init__(self, notional=100_000_000.0, coupon=0.060, base_yield=0.060):
        self.notional = notional
        self.coupon = coupon
        self.base_yield = base_yield

    def price(self, y):
        # Non-callable benchmark bond (positive convexity)
        p_nc = 100.0 * (1.0 - 5.5 * (y - self.base_yield) + 14.0 * (y - self.base_yield)**2)
        # Embedded American Prepayment Call Option
        incentive = self.coupon - y
        opt_val = 15.0 / (1.0 + np.exp(-150.0 * (incentive - 0.006)))
        return p_nc - opt_val

    def get_greeks(self, y, dy=0.0001):
        p0 = self.price(y)
        p_up = self.price(y + dy)
        p_down = self.price(y - dy)
        dur = (p_down - p_up) / (2.0 * p0 * dy)
        cvx = (p_down + p_up - 2.0 * p0) / (p0 * (dy**2))
        dv01 = (p_down - p_up) / (2.0 * dy) * 0.0001 * (self.notional / 100.0)
        return {'price': p0, 'duration': dur, 'convexity': cvx, 'dv01': dv01}
```

```python

class SwaptionHedgeInstrument:
    """
    Models an OTC SOFR Receiver/Payer Swaption for synthetic convexity replication
    .
    """
    def __init__(self, strike=0.060, maturity=10.0, vol=0.20, is_receiver=True):
        self.strike = strike
        self.maturity = maturity
        self.vol = vol
        self.is_receiver = is_receiver

    def price(self, y):
        annuity = (1.0 - (1.0 + y)**(-self.maturity)) / y
        sign = 1.0 if self.is_receiver else -1.0
        d = sign * (self.strike - y)
        sigma_t = max(0.001, self.vol * 0.01 * np.sqrt(1.0))
        d_norm = d / sigma_t
        val = (d * norm_cdf(d_norm) + sigma_t * norm_pdf(d_norm)) * annuity *
    100.0
        return float(val)

    def get_greeks(self, y, dy=0.0001):
        p0 = self.price(y)
        p_up = self.price(y + dy)
        p_down = self.price(y - dy)
        dur = (p_down - p_up) / (2.0 * max(0.01, p0) * dy)
        cvx = (p_down + p_up - 2.0 * p0) / (max(0.01, p0) * (dy**2))
        dv01_per_m = (p_down - p_up) / (2.0 * dy) * 0.0001 * (1_000_000.0 / 100.0)
        return {'price': p0, 'duration': dur, 'convexity': cvx, 'dv01_per_m':
    dv01_per_m}

def run_prepayment_hedge_simulation():
    mbs = MortgagePrepaymentPool(notional=100_000_000.0, coupon=0.060, base_yield
    =0.060)
    g_mbs = mbs.get_greeks(0.060)

    # 10Y ATM Receiver Swaption (Strike = 6.0%)
    rec_swpn = SwaptionHedgeInstrument(strike=0.060, maturity=10.0, vol=0.20,
    is_receiver=True)
    g_swpn = rec_swpn.get_greeks(0.060)

    # 1. Neutralize Negative Convexity with Receiver Swaption:
    mbs_dollar_gamma = (g_mbs['convexity'] * g_mbs['price']) * (mbs.notional /
    100.0)
    swpn_dollar_gamma_per_m = (g_swpn['convexity'] * g_swpn['price']) * (1_000_000
    .0 / 100.0)
    swpn_notional = abs(mbs_dollar_gamma) / swpn_dollar_gamma_per_m * 1_000_000.0

    # 2. Neutralize Net Residual Delta with Linear Payer SOFR Swap:
    swap_dv01_per_m = 750.0  # 10Y swap duration ~7.5 years
    swpn_total_dv01 = (g_swpn['dv01_per_m'] / 1_000_000.0) * swpn_notional
    net_dv01_to_hedge = g_mbs['dv01'] + swpn_total_dv01
    swap_notional = (net_dv01_to_hedge / swap_dv01_per_m) * 1_000_000.0

    print("="*80)
    print("MORTGAGE PREPAYMENT OPTION & SYNTHETIC CONVEXITY HEDGING ENGINE")
    print("="*80)
    print(f"MBS Collateral Notional:     ${mbs.notional:,.2f}")
```

```python
    print(f"Base MBS Price:                ${g_mbs['price']:.2f} per $100 par")
    print(f"Base Effective Duration:       {g_mbs['duration']:.2f} years")
    print(f"Base Negative Convexity:       {g_mbs['convexity']:.1f} (Pronounced Negative Gamma)")
    print(f"Base Portfolio DV01:           ${g_mbs['dv01']:,.2f} per bp")
    print(f"\nSynthetic Multi-Asset Hedge Sizing:")
    print(f"1. Linear Payer SOFR Swap:     Notional = ${swap_notional:,.2f}")
    print(f"2. OTC Receiver Swaption:      Notional = ${swpn_notional:,.2f} (Gamma Immunization)")

    shocks = [-0.0100, -0.0050, 0.0, 0.0050, 0.0100]
    p_base = g_mbs['price']

    results = []
    for s in shocks:
        y = 0.060 + s
        p_mbs = mbs.price(y)
        mbs_pnl = (p_mbs - p_base) * (mbs.notional / 100.0)

        # Pure linear delta hedge P&L:
        linear_pnl = -(g_mbs['dv01'] / 0.0001) * s
        linear_net = mbs_pnl + linear_pnl

        # Synthetic overlay P&L:
        swap_pnl = -(swap_notional * 7.50 * s)
        swpn_pnl = (rec_swpn.price(y) - g_swpn['price']) * (swpn_notional / 100.0)
        synth_net = mbs_pnl + swap_pnl + swpn_pnl

        results.append({
            'Shock (bps)': int(s * 10000),
            'MBS P&L ($)': mbs_pnl,
            'Linear Net ($)': linear_net,
            'Synthetic Net ($)': synth_net,
            'Convexity Alpha ($)': synth_net - linear_net
        })

    df = pd.DataFrame(results)
    print("\n" + "="*80)
    print(f"{'Shock (bps)':<12} | {'MBS P&L ($)':<16} | {'Linear Net ($)':<16} | {'Synthetic Net ($)':<16} | {'Alpha ($)':<12}")
    print("-" * 80)
    for _, r in df.iterrows():
        print(f"{int(r['Shock (bps)']):+5d} bps   | ${r['MBS P&L ($)']:>14,.2f} | ${r['Linear Net ($)']:>14,.2f} | ${r['Synthetic Net ($)']:>14,.2f} | ${r['Convexity Alpha ($)']:>10,.2f}")
    print("="*80)
    print("\nAnalytical Insight:")
    print("Under the Linear Delta Hedge, negative convexity inflicts severe capital bleed")
    print("during rallies and extensions. The Synthetic Swaption Overlay provides positive")
    print("gamma, stabilizing portfolio equity value across interest rate shocks.")

if __name__ == "__main__":
    run_prepayment_hedge_simulation()
```

Listing 1: MBS Prepayment Option, Linear Slippage, and Swaption Overlay Replication Engine